Some remarks on black hole temperature and the second law of thermodynamics

Marco Scandurra *
University of Pisa
Dipartimento di Fisica
Via Buonarroti 2, I-56127 Pisa, Italy

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Abstract

I present a formulation of the second law of thermodynamics in the presence of black holes which makes use of the efficiency of an ideal machine extracting heat cyclically from a black hole. The Carnot coefficient is found and it is shown to be a simple function of the mass.

Black holes are known to radiate energy thermally with a temperature given by a simple function of the mass of the black hole[1]. In the case of an uncharged, non rotating black hole the temperature is given by

$$T(M) = \frac{\hbar c^3}{8\pi k_B G M} \ . \tag{1}$$

The radiation has a quantum origin: it arises from the distortion of the vacuum fluctuations of the quantum fields near the event horizon [1, 2]. Classical black holes are found to obey ordinary laws of thermodynamics [3]. Furthermore, there are reasons to believe that radiating black holes obey a generalized second law of thermodynamics, stating that the sum of the entropy of the black hole $\frac{1}{4}A$ and the ordinary entropy outside it S never decreases [4]. The role of the entropy inside a black hole is played by the event horizon area A.

Since a black hole is fully comparable to a black body emitting real radiation, one could imagine that a body at a lower temperature located in its vicinity could absorb part of this radiation. However, if our goal is to extract energy from the black hole, we would find that only a limited quantity of energy available for doing work could be obtained by this method. The impossibility of extracting

*e-mail: mscandurra@bert.uchicago.edu; scandurr@df.unipi.it

energy from a single heat bath is a strict consequence of the second law of thermodynamics.

In this note I would like to show how the second law of thermodynamics in the context of black hole physics can be formulated[1] by means of the Carnot coefficient η, which is a well known parameter in classical thermodynamics expressing the efficiency of a thermal machine exploiting two heat reservoirs.

In the formulation by Thomson, the second law of thermodynamics states that a machine can not extract energy cyclically from a heat reservoir without changing anything in the external environment. At least two heat reservoirs at temperatures T_1 and T_2 are needed in order to transform heat into work. Then a machine can mine energy cyclically from the hotter reservoir T_1 by releasing a part of it to the cooler one T_2. The maximum possible efficiency of a cycle is given by the Carnot coefficient

$$\eta = \frac{T_1 - T_2}{T_1} \ . \tag{2}$$

If Hawking radiation has to be considered as real and if the generalized second law of thermodynamics holds, then a coefficient η_{BH} should exist and it should express the efficiency of an ideal machine which extracts energy cyclically from a black hole. More exactly, we imagine a machine which exploits two black holes with masses M_1 and M_2. The machine takes heat from the black hole with smaller mass M_1 and releases a part of it to the black hole with larger mass M_2. Only a fraction of the extracted energy can be transformed into work. Using the expression for the temperature given in (1) for masses M_1 and M_2 and inserting it into formula (2) we obtain

$$\eta_{BH} = \frac{M_2 - M_1}{M_2} \ < \ 1 \ . \tag{3}$$

This has to be considered as the Carnot coefficient in black hole mechanics. The fact that η_{BH} is always smaller than one is equivalent to asserting that the entropy of the system can never decrease. In fact, if we could extract energy from a single black hole we would decrease its entropy (though we would increase its temperature), without causing any other modification in the vicinity.

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References

[1] It is in fact only a different formulation, not a proof of its validity

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